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Instabilities of flows due to rotating disks: preface

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Abstract There are many flows driven by the rotation of one, or more, disks, and this paper is concerned with the instabilities of such flows, and their laminar–turbulent transition. The original, and most studied, rotating-disk flow is the von Kármán swirling flow produced by an infinite rotating disk in an otherwise still fluid. This flow shares many stability characteristics with three-dimensional boundary layers of engineering interest over aerofoils; most notably, the cross-flow instability giving rise to stationary cross-flow vortices. Various basic flows produced by rotating disks, and their stability, are reviewed, and motivations for assembling this special issue dedicated to the instabilities of rotating-disk flows are presented. The papers appearing in this special issue are discussed and related to major research themes in the field, and to one-another.

Keywords Hydrodynamic stability · Laminar–turbulent transition · Rotating-disk flows

1 Introduction

The flow produced by a disk rotating in an otherwise undisturbed incompressible viscous fluid has been central to the development of the hydrodynamic stability theory of three-dimensional boundary layers. Fluid near the surface of the disk is dragged by viscous stresses in the circumferential direction, and then thrown radially outwards by centrifugal effects. The rotating disk thus acts as a centrifugal fan, and there is an axial flow towards the disk surface that replaces the fluid thrown radially outwards.

There are a number of reasons why this simple basic flow has captured the attention of many of the finest fluid-dynamics researchers, both theoretical and experimental, over the last 50 years or so, why it continues to do so, and why it has had such an important influence on the development of our understanding of laminar–turbulent transition in three-dimensional boundary layers. In this Introduction we can only outline some of these reasons, and some of the historical developments, but see [1] for a recent review.

It was von Kármán [2] who discovered the similarity solution that describes the axisymmetric steady laminar basic flow for an infinite disk rotating at constant angular velocity in otherwise still fluid. Far from the axis of rotation this solution has boundary-layer characteristics, but no boundary-layer approximations

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are made; it is an exact solution of the Navier–Stokes equation. Compared to most three-dimensional boundary layers, which require numerical solution of the boundary-layer equations, von Kármán's solution is very conveniently obtained. Furthermore, a laminar three-dimensional boundary layer can be realized in an experiment over a rotating disk more easily than over a swept wing because no wind-tunnel is required, and the flow is not subject to leading-edge contamination from the body on which the swept wing is mounted.

These advantages were recognized, and exploited, in the pioneering paper by Gregory, Stuart and Walker [3], in which both experimental and theoretical stability results were presented. They noted that transition on swept wings occurs nearer to the leading edge than on unswept wings, and that flow visualizations reveal the existence of stationary vortices on swept wings. These stationary vortices were then demonstrated in an experiment on the rotating-disk boundary layer, and studied in detail for that flow. The linear stability theory for three-dimensional boundary layers was also developed in this paper. In particular, it is shown that in the inviscid limit the stability is governed by the familiar two-dimensional Rayleigh equation [4] with an effective basic velocity profile that is resolved in the direction of the wave-vector (when both wavenumber components are real). There are two important consequences of this observation. First, in this limit, streamline curvature and Coriolis terms are negligible. Therefore, conclusions concerning stability results in the inviscid regime for the rotating-disk boundary layer will be applicable to wider classes of boundary-layer flows and are not influenced by the rotation of the disk, or by the cylindrical geometry, but only by the cross-flow structure of the boundary layer. Second, the effective basic velocity profile depends continuously on a parameter (the wave-angle), giving a variety of basic flow profiles that exhibit a rich range of stability characteristics: there are monotonic profiles, inflexional profiles, profiles with turning points, wall-jet profiles with weak external counter- and co-flow, a profile with zero wall-shear, and a profile with inflexion point with zero basic velocity. This latter profile produces neutral stationary vortices, and was found to make a good quantitative prediction for the wave-angle of the stationary vortices observed in the experiment (but the wavenumber was significantly over-predicted, the discrepancy being attributed to viscous effects).

In summary, the rotating-disk boundary layer is conveniently established experimentally, it is conveniently described theoretically, and its rich stability characteristics, including the famous 'cross-flow' stationary vortices, carry over to many other three-dimensional boundary layers too. All of these points were made in Gregory, Stuart and Walker's paper, and indeed our understanding of the instabilities of this flow has continued to be informed by the fruitful interaction between experimental and theoretical investigations over the last 50 years or so (with, of course, numerical experiments becoming increasingly important too). This special issue reflects this tradition of interaction between theory and experiment in this subject, and a number of experimental papers have been included, in addition to theoretical papers, in the hope that mathematical readers of this journal can be inspired by the latest experimental findings, and can observe in some cases the particularly close interplay between theory, experiment and simulation.

The rotating-disk boundary layer, however, is not only a handy model for aerospace problems associated with swept wings on aircraft and turbine blades in jet engines etc. It is also a member of an important family of rotating flows that all have a similarity structure, in which both fluid and disk can rotate [5]. Other special cases of this family of flows are the Bödewadt layer where the disk is stationary and the fluid rotates, and the Ekman layer, where disk and fluid co-rotate at almost the same angular velocity. The similarity structure also persists when there is mass flux through the disk surface (suction/blowing), when there is an axial stagnation flow directed towards the disk surface and when there is a magnetic field parallel to the axis of rotation. The rotating-disk flow is also a special case of the family of rotating-cone flows when the vertex half-angle is $\pi/2$. Many aspects of the behaviour of disturbances in the classic von Kármán rotating-disk flow have their counterparts in these other flows too.

Even setting aside these connections to related flows, the rotating-disk boundary layer is a flow of interest in its own right. It has been shown that there are also stationary vortices in the high-Reynolds-number limit with long wavelengths compared to the boundary-layer thickness that are fixed by a balance between viscous and Coriolis forces and that cannot be determined by an inviscid theory [6]. For these waves the effective resolved basic velocity profile has zero wall-shear. These modes make up the lower branch of the neutral curve for stationary vortices, while the inviscid inflexional modes [3] make up the upper branch of this neutral curve. Work has also been done on the travelling waves in this problem, and they can also have viscous or inviscid origins. Other areas of research include secondary instabilities of the stationary vortices, the nonlinear evolution of disturbances and the receptivity of disturbances from the freestream into the boundary layer.

Another important result is that the local stability characteristics change at a well-defined critical local Reynolds number from convectively unstable, with disturbances only propagating and growing in the outward radial direction (as might be expected from the outward radial component of the basic flow), to absolutely unstable, with disturbances growing in time at fixed points over the disk [7]. Intriguingly, this Reynolds number corresponds closely to that at which the laminar flow becomes turbulent in experiments. However, the relationship between this qualitative change in local stability, and laminar–turbulent transition, is not straightforward, and will be discussed further in Sect. 2 since it has been addressed in several papers in this special issue.

In fact, my own interest in the stability of the rotating-disk boundary layer arose from questions concerning its absolute instability. Like the neutral curve for stationary vortices, the neutral curve for absolute instability has an inviscid upper-branch and a viscous lower-branch, but the 'pinch-point' [8] that controls absolute instability involves the pinching of different modes along each branch [9]. A long-wave inviscid theory for the pinch-point reveals that it can become asymptotically close to the imaginary axis of the radial wavenumber plane where branch-cuts are placed to exclude disturbances that grow exponentially with distance from the disk [10]. In this limit the effective resolved basic profile is a wall-jet with weak external counter-flow. The physical consequence of the proximity of the pinch-point to the continuous spectrum associated with the branch-cut, is the appearance of disturbances that do, nonetheless, propagate and grow exponentially in the wall-normal direction [11]. This behaviour of the pinch-point has been observed in the dispersion relations of other flows, including models for swirling jets used in vortex-breakdown studies, so this type of wall-normal instability discovered in the rotating-disk flow can occur in other flows too.

Moreover, there are areas of practical application where flows produced by rotating disks are important. Of course, high-speed machinery often contains components that rotate quickly, and while flows like those on the rotating disk may arise, the detailed structure of that flow may not necessarily be important in the overall performance of the machine. But the flow produced in the gap between rotating disks can be important in the smooth operation of digital storage devices (e.g hard disk drives). In such flows the effects of confinement and finite geometry become important. See the review [12] for further discussion of swirling flows in finite geometry and their relation to the similarity solutions of the infinite disk in unbounded fluid. Several papers in this special issue are concerned with confined rotating-disk flows.

Finally, one can point out that the rotating disk provides a convenient experimental and theoretical context for the investigation of a variety of further physical phenomena. For example, studies on the effects of surface compliance on instability waves have been carried out on rotating disks and are a particular case of the larger field of fluid–structure interactions. Studies of the formation of ripple patterns in sandy beds by three-dimensional boundary layers have also been carried out in rotating-disk flows. A paper on this problem and on wall-compliance are also included in this special issue.

Rotating-disk flows thus act as a focus that has attracted researchers from different areas of fluid mechanics with diverse motivations. This special issue seeks to present a cross-section of contemporary research on instabilities of rotating-disk flows some 50 years after the subject was opened up by the work of Gregory, Stuart and Walker.

2 This special issue

The papers in this special issue might have been grouped by whether they are largely theoretical, or experimental, or numerical, in nature, but to emphasize the connections between these approaches they are grouped, where appropriate, by topic, and appear roughly in the order discussed in Sect. 1. In what follows, unless indicated otherwise, the flow referred to is the classic von Kármán rotating-disk boundary layer.

Gajjar [13] presents effects of nonlinearity on the upper-branch stationary vortices. The disturbances are described using matched asymptotic expansions in the large-Reynolds-number limit, and nonlinearity first becomes important in the critical layer, where it reduces the phase-shift of the linear problem. This leads to significant modifications to the wavenumber and wave-angle of neutral nonlinear stationary vortices. In particular, the wavenumber decreases, in qualitative agreement with experiment. Although Coriolis terms are negligible in the linear theory near the upper-branch, they are shown here to be important in the nonlinear critical layer.

There are three papers, numerical, theoretical and experimental, concerned with the absolute/convective nature of the instabilities. Competing mechanisms are proposed for the laminar–turbulent transition and the role of the local absolute instability is questioned.

Davies et al. [14] review their recent direct numerical simulations of linearized impulsive disturbances; it is shown that, when the full inhomogeneous basic flow is considered (i.e., without making the parallel-flow approximation used in [7]), disturbances no longer grow in time over fixed points on the disk. This shows that there is no unstable linear global mode, even where the flow is locally absolutely unstable. It is argued that this is due to the local dimensionless frequency of the absolute instability varying with radius, which leads to a destructive interference that cancels the temporal growth. This phenomenon is known in astrophysical problems as phase-mixing. The authors show how this behaviour can be reproduced using a linearized complex Ginzburg–Landau equation with spatially varying coefficients as a model.

Although these simulations show that the linear global mode is stable, Pier [15] argues that there exists a nonlinear global mode as soon as the flow becomes locally absolutely unstable. He then shows that this nonlinear global mode is absolutely unstable to secondary instabilities. Therefore, small-enough initial disturbances ultimately decay, but larger ones are expected to trigger a nonlinear mode which then breaks down by secondary absolute instability. There is strong inflexional inviscid convective instability upstream of the nonlinear global mode, and the simulations [14] show strong transient growth of the linear global mode, so Pier argues that the threshold amplitude for exciting the nonlinear global mode will be very low, and easily exceeded by background disturbances. It is proposed that this explains the self-sustained turbulence in experiments at Reynolds numbers close to the onset of local absolute instability. He then develops a control strategy based on applying forcing in the upstream convective region at frequencies designed to excite a nonlinear global mode that will replace the naturally selected global mode and that is less absoutely unstable to secondary perturbations and thereby delay the onset of transition.

Corke et al. [16] present results of an experiment in which a very-low-level background disturbance environment has been achieved. Low-amplitude impulsive disturbances are introduced just outside the boundary layer in the convectively unstable region, and the leading and trailing edges of the resulting wavepackets are located and followed as the disturbance propagates radially outwards. It is found that laminar flow can be maintained a significant distance into the absolutely unstable region, and that the trailing edge of the wavepacket does not drop to zero velocity when the absolutely unstable region is reached, but behaves more like the damped global linear mode [14]. The authors argue that, therefore, in clean experiments, the flow is effectively convectively unstable. Pier's mechanism would then only apply if the background disturbance level is large enough (which was perhaps the case in earlier experiments). Corke et al. also investigate nonlinear resonant triad interactions. A stationary vortex of given azimuthal wavenumber (chosen to be close to the most unstable according to linear theory) is forced by arranging a spiral of dots on the disk surface, and spectra analysed for evidence of resonant interaction with background disturbances. The interactions favour excitation of a low-wavenumber mode, and some qualitative evidence for such a mode near transition can be seen in visualizations in earlier papers.

But perhaps these last two papers are closer to one-another than they appear—maybe natural background disturbances, aided by resonant interaction, actually delay transition beyond the radius of absolute instability in the manner that Pier suggests could be achieved by introducing appropriate external forcing, i.e., natural amplification removes the need for external forcing and the boundary layer controls itself...? We look forward to further research aimed at clarifying these issues.

Attention turns now to confined rotating-disk flows. Hewitt and Hazel [17] directly address the question of the connection between the similarity solution of the unbounded problem, and flows realized between counter-rotating disks of finite radius and aspect ratio. Numerical solutions to the steady axisymmetric Navier–Stokes equations in finite domains are obtained with several edge conditions, one of which is consistent with the similarity solution. Although the midplane symmetry-breaking bifurcation of the similarity solution also occurs in finite-domain calculations, the effect of finite radius, and the particular choice of boundary condition at the disk edge, has a dramatic effect on the location of the bifurcation point in parameter space, even when the aspect ratio is relatively large. Nonetheless, the symmetric similarity solution is found to be an excellent predictor for the flow.

Le Gal et al. [18] present results of experiments on the laminar–turbulent transition of the flow in the gap between a rotating disk close to a stationary disk. In this limit, far from the axis of rotation, the flow approaches plane-Couette flow, which, famously, is stable to infinitesimal disturbances, but can sustain finite-amplitude turbulent disturbances at large-enough Reynolds numbers. However, in torsional Couette flow there is a secondary outward radial flow near the rotating disk, as in the von Kármán layer, and an inward radial flow near the stationary disk, as in the Bödewadt layer. It is shown that turbulent spots spiral inwards towards the axis of rotation, and so they experience a decreasing local Reynolds number, and relaminarize at a critical radius. Study of these turbulent spots, using flow visualization techniques, has allowed an amplitude–threshold curve for self-sustained disturbances as a function of Reynolds number to be deduced. The critical Reynolds number below which all spots decay is $\mathrm{Re}_c = 2140$, and the amplitude threshold above which spots exist is found to scale as $(\mathrm{Re} - \mathrm{Re}_c)^{-3/2}$, which is consistent with earlier experiments on plane Couette flow.

Carpenter and Thomas [19] review research on the stability of flows over compliant rotating disks. Compliant surfaces have the potential to stabilize a flow, and the rotating disk provides a convenient case study for three-dimensional boundary layers. Much earlier work was focused on successfully suppressing disturbances in two-dimensional boundary layers, but it is not clear a priori whether the cross-flow instability mode can be suppressed simultaneously with the viscous modes, and without exciting hydro-elastic instabilities. The evidence from experimental, numerical and theoretical studies is discussed, including the effects of compliance on viscous, and inviscid, stationary, and travelling, vortices and the local absolute instability. Wall-characteristics can be tuned to substantially stabilize the inviscid instability and absolute instability, but tend to destabilize the viscous instabilities. It may be that more-sophisticated layered compliant surfaces can help, as is the case in two-dimensional boundary layers, but the simultaneous stabilization of all the modes of instability remains a challenge. Nonetheless, it seems progress can be made, especially if the dominant mechanisms driving transition can be established and then targeted.

Thomas and Zoueshtiagh [20] review their earlier results where they showed that the presence of a sandy bed produces a new type of spiral pattern. New visualizations are also presented showing that these new patterns are in addition to, and co-exist with, the one discovered by Gregory, Stuart and Walker. The experiments on sandy beds allow von Kármán and Bödewadt flows to be investigated during spin-up and spin-down, respectively, and also Ekman-type flows when the angular velocity of the bed is adjusted after spin-up has been completed. Experimental results for the new spiral pattern are also compared with numerical simulations, and it is found that the simulated patterns agree well with the experiment, even for very simple particle-moving rules in the simulation. In fact, the classic rotating-disk vortex pattern (wave-angle and wavenumber) is remarkably insensitive to the modified boundary condition despite the flow-induced granular motion, and deformed surface of the bed. However, with the sandy bed they appear at much lower Reynolds numbers than for a rigid disk. The Reynolds number for transition to turbulence is also much reduced. The sandy bed presents a surface roughness to the flow, but this is shown not to be sufficient to reduce the transitional Reynolds number by the amount seen in the experiments on sandy beds, i.e., the looseness of the bed is important in reducing the transition Reynolds number, though the mechanism remains obscure. Nonetheless, seeking an understanding of how the sandy bed destabilizes the inviscid vortices would seem to be an important step towards resolving these questions.

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